

- Matrices are arrays of numbers.
- Suppose  $A$  is an  $m \times n$  matrix (i.e.  $m$  rows and  $n$  columns).

$$\circ \quad A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

- $m$  and  $n$  are the dimensions of the matrix.

- **Square Matrix**

- $m = n$  Same number of rows as columns.

- **Determinant:**  $\det(A) = |A|$

- $2 \times 2$  matrix  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ :  $\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

- All other determinants can be simplified into determinants of  $2 \times 2$  matrices

- Example:  $3 \times 3$  matrix  $A$ .

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

- Remember to alternate the signs

- Represents signed volume inside matrix

- **Trace:** Sum of diagonal entries.  $\text{tr}(A) = a_{11} + a_{22} + \dots + a_{nn}$

- **Symmetric Matrix:** A square matrix such that  $A = A^T$

- **Orthogonal Matrix:** A square matrix such that  $A^{-1} = A^T$ .

- **Zero Matrix:** A matrix with entries of only 0

- Example:  $3 \times 2$  zero matrix  $0_{3,2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

- Any matrix multiplied by a zero matrix is another zero matrix.

- **Identity Matrix:** A square matrix with entries of 1 on its diagonal and 0 elsewhere

- Example:  $3 \times 3$  identity matrix  $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ .

- Any matrix multiplied by the identity matrix is itself.